Power Systems Planning and Integration of Renewables
Individual Decisions and Social Optimality

Alberto J. Lamadrid

Research Proposal
Outline

1. Sequentiality of Decisions with Varying Levels of Wind Penetration
   - Recapitulating what has been done
   - Current Work
   - Further Work Proposed

2. Dynamic Optimization with Successive Information Updates
   - Motivation, The Objective Function & Time Horizon
   - Information Updates
   - Further Work to be done

3. Individual Household Optimization
   - Objectives, Models Considered
   - Problem Formulation
   - Two Related Problems: Optimal Use and Optimal Replacement of a battery
   - Proposal for Further Work
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Linking hour to hour optimization

\[
\min_{G_{ik}, R_{ik}, \text{LNS}_{jk}} \sum_{k=0}^{K} p_k \left\{ \sum_{i=1}^{I} \left[ C_{G_i}(G_{ik}) + R_i^+(G_{ik} - G_{i0}^{t-1}) + R_i^-(G_{i0}^{t-1} - G_{ik}) \right] \right\} + \sum_{j=1}^{J} \text{VOLL}_{j} \text{LNS}(G_k, R_k)_{jk} \\
+ \sum_{i=1}^{I} \left[ C_R(R_i^+) + C_R(R_i^-) \right]
\]

Subject to meeting Load and all of the nonlinear AC constraints of the network.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k = 0, 1, \ldots, K )</td>
<td>Contingencies in the system</td>
</tr>
<tr>
<td>( i = 0, 1, \ldots, I )</td>
<td>Generators</td>
</tr>
<tr>
<td>( j = 0, 1, \ldots, J )</td>
<td>Loads</td>
</tr>
<tr>
<td>( p_k )</td>
<td>Probability of contingency ( k ) occurring</td>
</tr>
<tr>
<td>( G_i )</td>
<td>Quantity of active power generated (MVA)</td>
</tr>
<tr>
<td>( C_{G_i}(G_i) )</td>
<td>Cost of generating ( G_i ) MVA</td>
</tr>
<tr>
<td>( R_i^+(G_{ik} - G_{i0}^{t-1}) )</td>
<td>Cost of increasing generation from previous hour</td>
</tr>
<tr>
<td>( R_i^-(G_{i0}^{t-1} - G_{ik}) )</td>
<td>Cost of decreasing generation from previous hour</td>
</tr>
<tr>
<td>( \text{VOLL}_j )</td>
<td>Value of Lost Load, ($)</td>
</tr>
<tr>
<td>( \text{LNS}(G, R)_{jk} )</td>
<td>Load Not Served (MWh)</td>
</tr>
<tr>
<td>( R_i^+ )</td>
<td>((\max(G_{ik} - G_{i0}))^+), Up reserves quantity (MW)</td>
</tr>
<tr>
<td>( C_R(R_i^+) )</td>
<td>Cost of providing ( R_i^+ ) MW of up Reserves</td>
</tr>
<tr>
<td>( R_i^- )</td>
<td>((G_{i0} - \min(G_{ik}))^+), Down reserves quantity (MW)</td>
</tr>
<tr>
<td>( C_R(R_i^-) )</td>
<td>Cost of providing ( R_i^- ) MW of down Reserves</td>
</tr>
</tbody>
</table>
Recapitulating what has been done

Some of the setup considered

**Setup ramping costs**

No dynamic optimization, but linkage between results over 24 hours [Mount et al. (2010)]{#mount2010}. For every hour, a two-stage optimization problem was solved [Zimmerman, Murillo-Sanchez, and Thomas (2009)].

- First stage (hour-ahead), the dispatches for the next time period \((t + 1)\) were determined.
- Second stage (real-time), wind realization is known \(\rightarrow\) dispatches for the present time period \((t + 1)\) were determined with reserves from results of first stage.
- Outputs of each hour were interlinked \(\Rightarrow\) set second-stage dispatches for hour \(t\) as initial conditions for the dispatch in hour \(t + 1\).
- Any deviations above or below previous hour dispatch priced according to the ability of generators to move from their current operating point.

*With this setup, there are some questions that need to be addressed:*

1. What are the optimal ramping costs adopted?
2. How to better reflect environmental costs to the system?
Fuel Utilization per hour of day

### Case 1
- **Graph 1:** Fuel Utilization per hour of day, showing the dispatch per fuel type (MW) for each hour of the day for Case 1.
- **Graph 2:** Fuel Utilization per hour of day, Case 1n, showing the dispatch per fuel type (MW) for each hour of the day for Case 1n.

### Case 2
- **Graph 3:** Fuel Utilization per hour of day, Case 2, showing the dispatch per fuel type (MW) for each hour of the day for Case 2.
- **Graph 4:** Fuel Utilization per hour of day, Case 2n, showing the dispatch per fuel type (MW) for each hour of the day for Case 2n.

**Legend:**
- Wind
- Oil
- GCT
- CC Gas
- Coal
- NHR

**Hour of the day:**
- 1 to 24

**Dispatch per fuel type (MW):**
- 0 to 200

**Note:** The graphs illustrate the sequentiality of decisions with varying levels of wind penetration and recapitulate what has been done.
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Current work underway

1. Use of a new solver for the first stage (see description between solvers).

2. Replicate past results and analyze the effects of better storage modeling.

Differences with previous work done:

<table>
<thead>
<tr>
<th></th>
<th>First stage solver</th>
<th>Second stage solver</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reserves</td>
<td>Endogenously found (vs. Restricted)</td>
<td>Constant and provided (e.g. from first stage solver)</td>
</tr>
<tr>
<td>Contract power ($P_c$)</td>
<td>Endogenously found (vs. Restricted)</td>
<td>Constant and provided</td>
</tr>
</tbody>
</table>
Fuel Utilization per hour of day

**Case 4**

- Wind
- Oil
- GCT
- CC Gas
- Coal
- NHR

**Case 7**

- Wind
- Oil
- GCT
- CC Gas
- Coal
- NHR

**Case 10**

- Wind
- Oil
- GCT
- CC Gas
- Coal
- NHR

**Case 11**

- Wind
- Oil
- GCT
- CC Gas
- Coal
- NHR
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Further Work to be done

1. Start using the NPCC system for sequential runs.

2. Replicate cases done with NPCC system:
   - No congestion case.
   - Storage.
   - Different locations of wind farms.
   - Load changes

3. Analyze the effects of load response, compare to transmission expansion results.

4. For the questions above posed:
   1. Do sensitivity analysis to ramping costs.
   2. Review work on coupling of gas turbines to wind outputs (e.g. [Peterson, Apt, and Whitacre(2010)])

5. Quadratic ramping costs, start using different time scales for different ramping services.

6. Explore effects of congestion rents (Surin’s work).
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Ancillary Services for Storage

Why Storage? Some Applications
▲ Voltage Control
▲ Frequency Regulation
▲ Reserves:
▲ Supplemental Reserve
▲ Replacement Reserve

Some Technologies.
★ Flywheel
★ Super Capacitors
★ Super Conducting Magnetic Storage
★ Pump Storage
★ Chemical Batteries.

Energy Storage Considerations

Power and Energy vs. duration of different ancillary services.

Characteristics of Technologies available.
Variables

<table>
<thead>
<tr>
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<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>Vector of bus voltages and angles</td>
</tr>
<tr>
<td>$p$</td>
<td>Vector of real and reactive outputs for generating units</td>
</tr>
<tr>
<td>$I$</td>
<td>Vector of binary variables indicating which of the $p$ units are de-committed from system</td>
</tr>
<tr>
<td>$e$</td>
<td>Vector of real (and possibly reactive) outputs for ESS units</td>
</tr>
<tr>
<td>$f(\cdot)$</td>
<td>Cost Function for real and reactive power</td>
</tr>
<tr>
<td>$C(\cdot)$</td>
<td>Startup and shutdown costs/benefits</td>
</tr>
<tr>
<td>$f_u(\cdot)$</td>
<td>Cost Function including ESS units, real (and possibly reactive power)</td>
</tr>
</tbody>
</table>
The general formulation for this problem is shown in (1).

\[
\min_{x, p, e, I} f(x, p) + C(p, I) + f_u(x, p, e) 
\]

subject to

\[
g(x, p, e) = 0 \quad (2)
\]
\[
h(x, p, e) \leq 0 \quad (3)
\]
\[
x_{\min} \leq x \leq x_{\max} \quad (4)
\]
\[
0 \leq p \leq p_{\max} \quad (5)
\]
\[
e_{\min} \leq e \leq e_{\max} \quad (6)
\]
\[
l \leq A \begin{bmatrix} x \\ p \\ e \end{bmatrix} \leq u \quad (7)
\]
Research Proposal

Dynamic Optimization with Successive Information Updates

Motivation, The Objective Function & Time Horizon

Multiperiod OPF

- Use system replication for each time period.
- Link periods by ramping constraints.

\[-r^i_g \leq p^i_t - p^i_{t-1} \leq r^i_g\]  \hspace{1cm} (8)
\[-r^i_e \leq p^i_t - p^i_{t-1} \leq r^i_e\]  \hspace{1cm} (9)

- Add constraints for ESS management

\[l^i_e \leq \sum_{t \leq T} p^i_t \cdot t \leq u^i_e, \forall i \in \mathcal{E}, t \in \mathcal{T}\]  \hspace{1cm} (10)
\[\sum_{t \in \mathcal{T}} p^i_t \cdot t = 0, \forall i \in \mathcal{E}\]  \hspace{1cm} (11)
\[u^i_e = s c^i_0 \cdot u^i_{\text{max},e}\]  \hspace{1cm} (12)
\[l^i_e = l^i_{\text{min},e} + u^i_e\]  \hspace{1cm} (13)
\[l^i_{\text{min},e} = -u^i_{\text{max},e}\]
**Multiperiod OPF**

- Use system replication for each time period.
- Link periods by ramping constraints.

\[
-r_g^i \leq p_t^i - p_{t-1}^i \leq r_g^i \\
-r_e^i \leq p_t^i - p_{t-1}^i \leq r_e^i
\]  

- Add constraints for ESS management

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\]  

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\]  

\[
u_e^i = sc_0^i \cdot u_{\text{max},e}^i
\]  

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l_e^i = l_{\text{min},e}^i + u_e^i
\]  

\[
l_{\text{min},e}^i = -u_{\text{max},e}^i
\]
## Multiperiod OPF

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  -r^i_e \leq p_t^i - p_{t-1}^i \leq r^i_e
  \]  

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  \]
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Types of generation

Setup of Runs

- Penalization for load shedding as per user defined parameters
- All run for first stage is done as a day ahead.
  1. Start with best forecast available.
  2. Policy is changed hour to hour with new information (wind forecast, load forecast).
  3. Testing of ESS done in a single location (Urban area).
- Different solvers used - no comparison done on performance of each one.
- All units committed (I, vector of ones).

<table>
<thead>
<tr>
<th>Fuel</th>
<th>Cost($/MW)</th>
<th>Gen. Avail (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil</td>
<td>(p) 95</td>
<td>65</td>
</tr>
<tr>
<td>GCT</td>
<td>(p) 80</td>
<td>45</td>
</tr>
<tr>
<td>CC Gas</td>
<td>(s) 55</td>
<td>40</td>
</tr>
<tr>
<td>NHR</td>
<td>(s) 5</td>
<td>65</td>
</tr>
<tr>
<td>Coal</td>
<td>(b) 25</td>
<td>70</td>
</tr>
<tr>
<td>NHR</td>
<td>(b) 5</td>
<td>50</td>
</tr>
</tbody>
</table>
Results from run, starting at 7AM and on

Example (Wind Forecasts)

The wind forecasts are provided for seven hours of the day, each hour updating the most recent information for all the optimization horizon, shifting one hour and adding one extra hour.

\[
W_{fcst} = \begin{bmatrix}
  f(7) & f(8) & \ldots & f(13) \\
  f(8) & f(9) & \ldots & f(14) \\
  \vdots \\
  f(6) & f(7) & \ldots & f(12) 
\end{bmatrix} \quad (15)
\]
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Further Work to be done

- So far, results are for a local optimum.
- Explore global optimizers in the set.
- Improve on current Unit de-commitment algorithm and startup/shutdown cost of units.
- Explore ramping payments according to capability of the units in the system.
- Analyze effects of several units in the system.
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Model for Individual Agent Problem

Three main problems formulated:

1. Energy shifting → Behavioral change mostly.
2. Energy Storage → Technology adoption mostly.
3. Combination of problems above

Assumptions/notes

1. Limited information for individual consumer’s decision. Prices provided to consumer (smart grid).
2. Consumer is a price taker (very important).
3. Initial problem formulated is a linear program (similar to transportation model in OR).
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Research Proposal
Individual Household Optimization
Problem Formulation
## Definition of Variables

**Table:** Variables and terms for Consumer’s problem

<table>
<thead>
<tr>
<th>Variable/Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_{\tau t}^1$</td>
<td>Load to be shifted from hour $t$ to hour $\tau$.</td>
</tr>
<tr>
<td>$e_{ij}^2$</td>
<td>Energy stored in hour $t$ to be used in hour $\tau$.</td>
</tr>
<tr>
<td>$P_{W_t}^{load}$</td>
<td>Load to be served for period $t$. (demand, MW)</td>
</tr>
<tr>
<td>$c_d^t(\cdot)$</td>
<td>Cost of serving the demand at period $t$.</td>
</tr>
<tr>
<td>$c_{s,p}^t(\cdot)$</td>
<td>Cost of shifting load/transferring capacity to period $t$.</td>
</tr>
<tr>
<td>$a_t$</td>
<td>Fraction of load that can be shifted from period $t$.</td>
</tr>
<tr>
<td>$e_{t}^{min}$</td>
<td>Lower limit for load at period $t$.</td>
</tr>
<tr>
<td>$e_{t}^{max}$</td>
<td>Upper limit for load at period $t$.</td>
</tr>
<tr>
<td>$ll_t$</td>
<td>Lower limit at period $t$ for energy storage.</td>
</tr>
<tr>
<td>$ul_t$</td>
<td>Upper limit at period $t$ for energy storage.</td>
</tr>
<tr>
<td>$b_t(\cdot)$</td>
<td>Personal benefits of moving load/capacity to period $t$.</td>
</tr>
</tbody>
</table>
Individual Problem Formulation

\[
\begin{align*}
\max_{\epsilon_{\tau t}, p=1,2} & \quad B = \sum_{\tau}^{T} b_\tau \left( \sum_{t}^{T} e_{\tau t}^1 \right) + \sum_{\tau=2}^{T} b_\tau \left( \sum_{t=1}^{\tau-1} e_{\tau t}^2 \right) - \sum_{\tau=2}^{T} \sum_{t=1}^{\tau-1} c_{\tau t}^d (e_{\tau t}^2) - \sum_{\tau=2}^{T} c_{\tau}^d [(1 - a_\tau) P_{W_{\tau}}^{load}] \\
& \quad - \sum_{\tau}^{T} \sum_{t}^{T} c_{\tau t}^d (e_{\tau t}^1) + \sum_{\tau=2}^{T} \sum_{t=1}^{\tau-1} c_{\tau t}^d (e_{\tau t}^2) - \sum_{\tau}^{T} \sum_{t}^{T} c_{\tau t}^s p (e_{\tau t}^1) - \sum_{\tau=2}^{T} \sum_{t=1}^{\tau-1} c_{\tau t}^s p (e_{\tau t}^2)
\end{align*}
\]

s.t.

L. Shift constraints

\[
\sum_{\tau}^{T} e_{\tau t}^1 = a_t P_{W_{t}}^{load}, \text{ for } t = 1, \ldots, T.
\]

Hourly Limits

\[
e_{\tau}^{\min} \leq (1 - a_\tau) P_{W_{\tau}}^{load} + \sum_{t}^{T} e_{\tau t}^1 \leq e_{\tau}^{\max}, \text{ for } \tau = 1, \ldots, T
\]

ESS Limits

\[
ll_{\tau} \leq \sum_{t}^{\tau-1} e_{\tau t}^2 \leq ul_{\tau}, \text{ for } \tau = 2, \ldots, T
\]
Initial Model will assume household as a price taker → Linear Program.

Demand of Energy Normalized for New York State

Load profiles for Energy usage from a typical household in the UK.

Energy and Power density for different Technologies

Observed prices in New York City, 2008 (NYISO).
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Two related problems: Optimal ESS Use

**Objective:** If a household has access to an Energy Storage System (ESS), how to optimally use it?

Initial approach: assume Utility can be expressed as an analytical function. Neglect Transversality condition (for now).

<table>
<thead>
<tr>
<th>Variable/Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_t$</td>
<td>Consumption of the car (given in miles) in period $t$.</td>
</tr>
<tr>
<td>$B_t$</td>
<td>Capacity of the battery at period $t$.</td>
</tr>
<tr>
<td>$U(\cdot)$</td>
<td>Utility from consumption of the ESS.</td>
</tr>
<tr>
<td>$\delta$</td>
<td>instantaneous discount rate</td>
</tr>
</tbody>
</table>

Problem: \( \max_{c_t} \ W(c_t) = E_0 \left[ \int_{t=0}^{\infty} U(c_t) e^{-\delta t} \, dt \right] \)

\[ s.t. \]

Energy Storage Evolution

\[ dB = a(B_t, c_t, t) \, dt + b(B_t, c_t, t) \, dz \]

\[ B_0 > 0 \text{ given} \]
Solving the Problem

The Hamilton-Jacobi-Bellman (HJB) equation and Itô’s Lemma application for this problem is given in (18)

\[
\delta V(B) = \max_{c_t} \left[ U(c_t) + \frac{1}{dt} E_t dV(B) \right],
\]

\[
dV(B) = \frac{dV}{dB} dB + \frac{1}{2} \frac{d^2 V}{dB^2} dB^2
\]

\[
dV(B) = \left[ a(B_t, c_t, t) \frac{dV}{dB} + \frac{1}{2} b^2(B_t, c_t, t) \frac{d^2 V}{dB^2} \right] dt + b(B_t, c_t, t) \frac{dV}{dB} dz, \Rightarrow
\]

\[
\delta V(B) = \max_{c_t} \left[ U(c_t) + a(B_t, c_t, t) \frac{dV}{dB} + \frac{1}{2} b^2(B_t, c_t, t) \frac{d^2 V}{dB^2} \right]
\]

(18)

The FOC for this problem:

\[
\frac{dU(c_t)}{dc_t} + \frac{da(B_t, c_t, t)}{dc_t} \frac{dV}{dB} + b(B_t, c_t, t) \frac{db(B_t, c_t, t)}{dc_t} \frac{d^2 V}{dB^2} = 0
\]
Two related problems: Optimal Replacement

Optimal feedback policy \((c = \phi(B))\) obtained according to functions describing \(U(\cdot), a(\cdot), b(\cdot).\)

\[
\delta V(B) = U(\phi(B)) + a(B_t, \phi(B), t) \frac{dV}{dB} + \frac{1}{2} b^2(B_t, \phi(B), t) \frac{d^2V}{dB^2}
\] (19)

**Objective:** If a household has access to a an Energy Storage System (ESS), how to optimally manage the replacement problem?

ESS is durable good that depreciates over time.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Content</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>Marginal Utility of quality ($ terms).</td>
</tr>
<tr>
<td>(c)</td>
<td>Replacement cost of the unit.</td>
</tr>
<tr>
<td>(m)</td>
<td>Marginal utility of money.</td>
</tr>
<tr>
<td>(de)</td>
<td>depreciation at each period of time.</td>
</tr>
<tr>
<td>(d)</td>
<td>discount factor</td>
</tr>
<tr>
<td>(B_t)</td>
<td>Binary variable: 1 (capital good is replaced), or 0.</td>
</tr>
<tr>
<td>(X_t)</td>
<td>quality level of good A at time (t)</td>
</tr>
<tr>
<td>(X_0)</td>
<td>level of quality of good replaced.</td>
</tr>
</tbody>
</table>
Optimal ESS Replacement

Assumptions

- When good is replaced, no leftover value for capital good. \((1 - B_t)\) factor.
- Since good is replaced in time \(t\), in period \(t + 1\) it will be depreciated already. \((X_0 - de)\)

\[
\max_{B_t \in \{0,1\}} u = \sum_{t=0}^{\infty} d^t [aX_t - cmB_t]
\]

s.t. \(X_{t+1} = (1 - B_t)(X_t - de) + B_t(X_0 - de)\)

\(X_0\) given

Solving the HJB equation:

\[
V(X_t) = \max_{B_t \in \{0,1\}} \left( aX_t - cmB_t + dV(X_{t+1}) \right)
\]

s.t. \(X_{t+1} = (1 - B_t)(X_t - de) + B_t(X_0 - de)\)

\(X_0\) given

\[
\Rightarrow V(X_t) = \max_{B_t \in \{0,1\}} \left[ aX_t - cmB_t + dV\left( (1 - B_t)(X_t - de) + B_t(X_0 - de) \right) \right]
\]
Outline

1. Sequentiality of Decisions with Varying Levels of Wind Penetration
   - Recapitulating what has been done
   - Current Work
   - Further Work Proposed

2. Dynamic Optimization with Successive Information Updates
   - Motivation, The Objective Function & Time Horizon
   - Information Updates
   - Further Work to be done

3. Individual Household Optimization
   - Objectives, Models Considered
   - Problem Formulation
   - Two Related Problems: Optimal Use and Optimal Replacement of a battery
   - Proposal for Further Work
Further Work

Example

Depending on location, usage profile changes (Urban/rural, climate, Behavior) Use Li-Ion technology, [EIA2010(2010), EAC(2008)], due to range and general application ranges.

- Improve on the ESS modeling [Kalhammer et al.(2007) Kalhammer, Kopf, Swan, Roan, and Walsh].
- Get better data for household profiles [Yao and Steemers(2005)].
- Aggregation of single household (linearity assumption).
- Create other scenarios for different possible consumer objective functions, e.g.:
  - Reduce household footprint.
  - Increase independence from grid usage.
  - Minimize electricity bill.
  - Analyze non-linear effects (Aggregation of different types of agents).
1. Continue work using SuperOPF, sequential runs and further explore the effects of ramping costs.
2. Analyze the effect of several ESS in the network.
3. Model the consumer behavior with the advent of the smart grid.

Outlet
- Unit commitment or use of startup/shutdown costs.
- Modeling of Vehicles and electrification of transportation.
- Global optimization.


