Secure Planning and Operations of Systems with Stochastic Sources, Energy Storage and Active Demand

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***Abstract*—This work presents a stochastic optimization framework for operations and planning of an electricity network as managed by an Independent System Operator. The objective is to maximize the total expected net benefits over the planning horizon, incorporating the costs and benefits of electricity consumption, generation, ancillary services, load-shedding, storage and load-shifting.**

The overall framework could be characterized as a secure, stochastic, combined unit commitment and AC optimal power flow problem, solving for an optimal state-dependent schedule over a pre-specified time horizon. Uncertainty is modeled to expose the scenarios that are critical for maintaining system security, while properly representing the stochastic cost. The optimal amount of locational reserves needed to cover a credible set of contingencies in each time period is determined, as well as load-following reserves required for ramping between time periods. The models for centrally-dispatched storage and time-flexible demands allow for optimal tradeoffs between arbitraging across time, mitigating uncertainty and covering contingencies.

This paper details the proposed problem formulation and outlines potential approaches to solving it. An implementation based on a DC power flow model solves systems of modest size and can be used to demonstrate the value of the proposed stochastic framework.

I. Introduction

RECENT developments in electric power systems toward the deployment of smart grid technologies are creating new challenges for system operators and driving the need for enhanced methods and tools for the efficient and secure planning and operation of the modern power grid. The desired ability to operate efficiently and reliably in the face of significant and growing uncertainty points toward a more explicitly stochastic approach. At the same time, a multi-period framework is indicated by the inclusion of technologies whose operational characteristics are coupled in the time dimension, such as centrally dispatchable grid-level storage and time-flexible demand. Improvements to efficiency and to fidelity of valuation and pricing call for more complete, accurate, integrated and co-optimized models. Furthermore, these models should be employed within a market structure that clears as frequently as possible, based on the best and most recent information available, in a sequence of consistent multi-stage problems. To this end, this work presents a stochastic optimization framework that attempts to simultaneously incorporate these various goals. While some discussion of solution algorithms and implementation is included, the primary focus is on the problem formulation.

This framework has evolved over time, with its origins tracing back to [1], and evolution in [2]–[4]. It shares some common features with the approach in [5], but incorporates many additional considerations. Unlike some common approaches, the stochastic program that defines the day-ahead stage of the decision process is solved without resorting, as in [6], to iterations in which feasibility is assessed post-factum and a proxy problem is re-solved with additional constraints if problems arise. This is necessary for the shadow prices to convey true sensitivity information. The uncertainty in wind realization is addressed by means of wind scenarios, each of which is assured of secure operation in light of a list of credible contingencies, but the transitions are modeled using scenario recombination from time period to time period as in [7] and unlike [8]–[10]. While this may result in a conservative level of procured reserves, it facilitates the levels of reliability to which customers have become accustomed and it makes the problem more manageable by avoiding a scenario tree explosion.

Many authors neglect the need for an accurate representation of the network [7], [8], [10]–[14], but here the locational aspect is addressed directly by including a full network representation. There are few works that include deferrable demand or flexible storage technology [14], [15], and those that do usually impose constraints that are to be met in an expected sense, or they provide little flexibility of scheduling in the day-ahead problem. The storage formulation in this work is particularly flexible yet it ensures feasibility, and in fact also allows for different ways of specifying deferrable demand and thermal storage. To date, there is seems to be no other formulation in the literature that provides this level of flexibility while considering the network.

The next section gives an overview of the proposed approach to the problem and conceptual description of the problem formulation, with the mathematical details presented in Section III. Section IV discusses the nature of the resulting optimization problem and some approaches to solving it. Several considerations regarding implementation and testing

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are covered in Section V, followed conclusions in Section VI.

II. Overview of Proposed Approach

The overall proposed framework could be characterized as a secure, stochastic version of a combined unit commitment (UC) and AC optimal power flow (OPF) problem. The objective is to maximize the total expected net benefits, that is, the total expected benefits of electricity consumption minus the total expected costs of supplying energy and ancillary services over a given forecast horizon. This takes into account involuntary load-shedding in contingencies as well as active demand-side bidding and load-shifting. This will be referred to as the stage 1 problem and corresponds loosely to the problem solved in a typical day-ahead market context.

The dynamic nature of system operations, along with the inclusion of technologies such as centrally-dispatched storage, implies the need for a multi-period approach to planning. Intertemporal constraints and costs must be incorporated to ensure that transitions from one operating point to the next are feasible and economical. These costs and constraints typically apply independently to individual units while spanning multiple periods. In addition, there are the standard static OPF constraints and costs applied across the system at a given point in time, including power balance equations and voltage, generation and line flow limts. These static constraints must also include a framework for handling the uncertainty inherent in the parameters describing the corresponding system conditions.

A. Two Types of Uncertainty

The uncertainty faced by a system operator can be viewed as falling into one of two categories. The first relates to low probability discrete events such as line, generator or other equipment failures, in other words, contingencies. Each such event can be represented as a modified version of a base system state along with an associated probability. The second category has to do with the limited knowledge about the values of future model parameters, such as nodal demand or wind energy production. This type of uncertainty can be described as a set of probability distributions for the uncertain parameters. In this case, the distributions can be approximated by a set of system states and associated probabilities, each with specific realizations for the uncertain parameters. The selection of the set of system states and probabilities that best represents the underlying probability distributions is clearly an important consideration. Putting these two types of uncertainty together, with the second providing a set of base states or scenarios, and the first adding a set of corresponding contingency states to each of the base states, results in a tree-like structure of system states and corresponding probabilities that approximates the uncertainty faced in any given period in the planning horizon. Fig. 1 illustrates the structure of a problem with two base scenarios, each with two contingencies, in each period of a three period horizon.

The structure of each period corresponds to the formulation in [4] with the addition of multiple base scenarios. Upward and downward contingency reserves are defined by the maximum

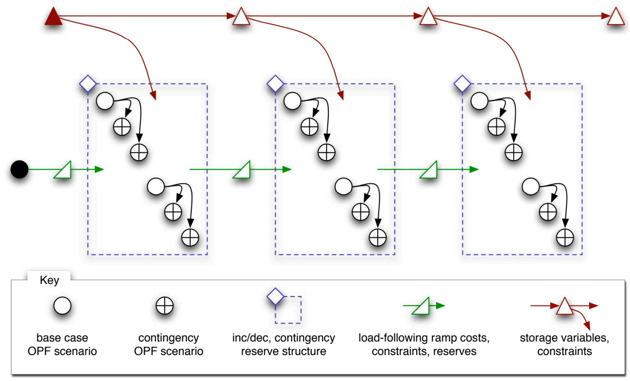

Fig. 1. Overall Problem Structure

deviations from a contracted reference output that are required to cover each of the system states, including the contingencies. In this way, the reserve requirements are locational and determined endogenously as a function of the uncertainty in the system conditions, not pre-specified as some zonal proxy for security computed from off-line studies. Within each scenario, physical ramp limits are imposed on each unit to ensure feasible transitions from the base to contingent states. Costs are imposed on reserves, and probability-weighted costs on both the injections and deviations from the contract in each state. Fig. 2 illustrates the structure for unit i at time t. This reserve formulation, along with the use of a full AC network model helps to avoid the obscuring or blocking of potential tradeoffs and distortion of prices that accompany the use of proxy constraints and DC network models.

When employing stochastic optimization to find an optimal operating plan, the goal is to maximize the total expected net benefit over all possible trajectories, taking into account both contingencies and forecasting uncertainty and ensuring the strict feasibility of each trajectory. However, for the sort of multi-period problem considered here, the set of all possible trajectories grows combinatorically with the number of base

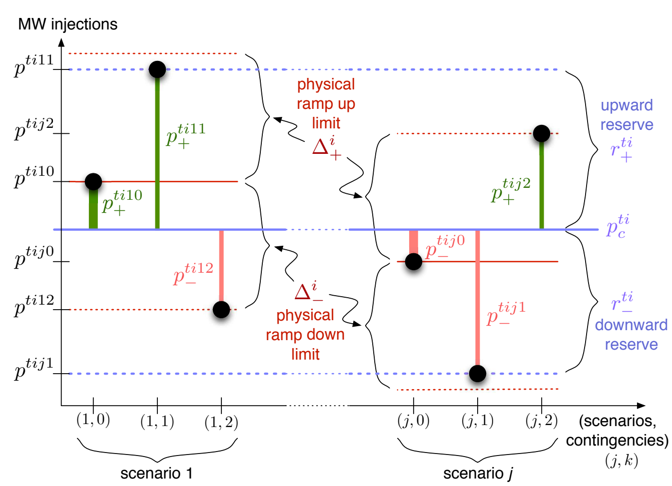

Fig. 2. Reserve structure for generator i in period t.

system states in each period and the number of time periods in the horizon, and linearly with the number of contingencies in each base state. Clearly, with more than a few base states, contingencies or periods the full problem quickly becomes intractable, calling for additional approximations.

One rather extreme approach is to use a traditional deterministic optimization over a single expected or forecasted trajectory, but this becomes unacceptable as uncertainty increases. Enforcing some sort of "expected feasibility" based on, for example, expected storage states and expected ramping requirements is not sufficient to ensure security of operation. Selecting a specific set of representative trajectories [13], on the other hand, has the benefit of allowing the enforcement of strict feasibility, but it requires a very large number of trajectories to capture the full range of possibilities.

The approach proposed in this work attempts to take into account a larger number of trajectories without sacrificing strict feasibility within a high-probability "central path". This is done by handling the two types of uncertainty mentioned above in different ways. It is assumed that contingencies are considered in an $N-1$ context. That is, the solution to the optimization will be secure for each considered contingency, but any actions following the event will have to be determined by solving a new problem. In other words, we are not planning for $N-2$ or projecting future states following a contingency. The base states in each period and the corresponding transitions between periods, on the other hand, define a central path or operating point envelope in which all transitions must be feasible and a Markovian transition probability matrix governs the propagation from period to period.

The transition along this central path from the set of base scenarios at time t to the set of base scenarios at time $t+1$ can happen in many different ways and each possible transition has an associated probability of occurrence, conditional on the actual realized scenario at time t. A simple model of this setting, based on a probability transition matrix that relates the vector of base case probabilities at time t to the vector of scenario probabilities at time $t+1$, is used in this formulation.

B. Load-following Ramping

The feasibility of this central path is ensured by applying a number of intertemporal constraints between adjacent time periods. The first of these are the critical load-following ramping constraints needed to guarantee sufficient ramping capability to follow rapidly changing system conditions, such as those encountered during morning load ramp-up or a sudden change in wind generation. At each hour, this capability is determined by two types of decisions: the type, number, capacity, location and ramp capability of the units that are online, and the actual scheduling of the pool of committed units. Load-following ramping limits are applied for each unit, constraining the corresponding change in dispatch for all transitions, that is, from each scenario j_1 in period t to each scenario j_2 in period $t+1$. This approach is a compromise between the goals of attaining security, accurately representing a stochastic program and keeping problem size manageable. In particular, these ramp constraints are covering for extreme variations that may be very rare in practice, particularly if the scenarios are chosen to represent the geographically multivariate tails of the wind output distribution in addition to the most plausible scenarios. But this worst-case scenario planning approach is consistent with secure operation philosophy.

In addition to the hard limits on upward and downward load-following ramp, the formulation includes probability-weighted ramping costs on each of the individual transitions as well as costs, representing market offers for a potential ramping reserve product, on the maximum up and down ramp capacities procured in the optimal schedule.

C. Storage

Storage and time-flexible loads can be controlled independently or centrally managed. The latter requires special treatment in the context of multiple scenarios per period. Since energy storage limits must be respected in every realization throughout the planning horizon, one modeling choice would be to include a single set of dispatch variables for storage units through all of the scenarios. This, of course, limits flexibility and eliminates, for example, the option to use the storage dispatch to offset variability of renewables.

In this model, a more flexible approach was used in which each storage unit has two optimization variables for each period, representing the lower and upper bounds on the amount of stored energy during that period. These variables are then constrained to ensure that all of the possible "charge" and "discharge" schedules associated with the central path system states are feasible with respect to the energy capacity limits of the storage unit. Thus, a lower bound on stored energy at the end of period t depends on the lower bound at time $t-1$ and the maximum depletion over the base scenarios at time t. An example where these bounds spread to the capacity limits of the unit is shown in Fig. 3 Here the diamonds represent the expected stored energy and the dashed lines correspond to the storage dispatches, which may or may not vary by scenario.

The amount of energy stored in a given unit at the end of period t is assumed to depend on three things: the initial stored

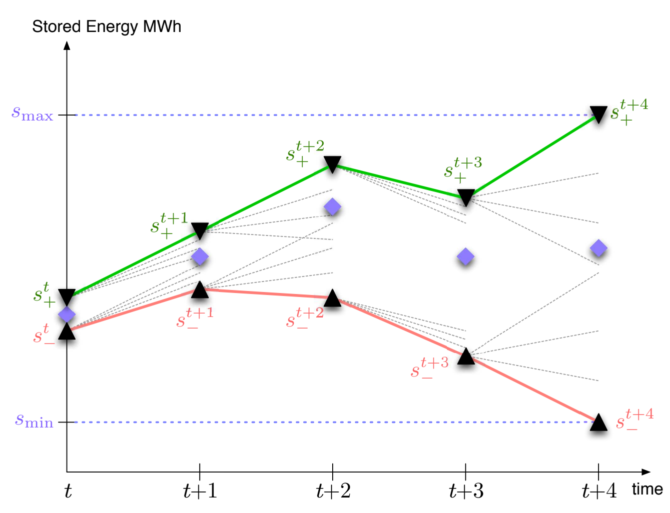

Fig. 3. Storage Formulation: lower and upper bound storage state variables.

energy at time $t = 0$, the dispatch of the unit in periods 1 through t (adjusted for charging and discharging efficiency losses) and a per-period energy loss equal to a loss factor times the average stored energy across the period.

In addition to the central path restrictions on storage, the use of stored energy in contingency states must also respect energy storage limits. This is subject to a factor α that specifies, for contingency cases, the fraction of the time slice that is spent in the base case operation before the contingency occurs.

Storage units can have leftover stored energy at the end of the horizon or at any other endpoint in the transition tree, such as in a contingency. Including the value of this expected quantity of energy in the objective function is a critically important feature when centrally optimizing the use of storage. In this framework, since the expected stored energy quantity at any state can be expressed as a linear combination of all of the charging and discharging injections, an appropriately weighted value is associated with each of the injections. Increases in this expected stored energy can actually be valued differently from decreases, and injections in contingency cases differently from those in base scenarios at the end of the horizon. Options are also included to set targets for the final expected stored energy or to constrain it to equal a floating initial value.

This formulation for storage accommodates several types of resources including traditional pumped-storage, batteries that do not require pre-specified multi-period charge-discharge trajectories, combined thermal storage and thermal load or aggregations of such individual resources. Even dispatchable loads with an energy consumption quota over a given horizon can be specified. A unique feature of this storage model is that it allows for optimal tradeoffs between using storage to arbitrage across time, mitigate uncertainty or cover contingencies.

D. Unit Commitment

Unit commitment decisions also have an intertemporal dependency in the form of minimum up- and down-times. In addition, these decisions introduce binary variables with associated startup and shutdown costs for each unit in each time period. This structure results in a single commitment schedule for the planning horizon that applies to all of the scenarios under consideration. As with other post-contingency actions, updates to this commitment schedule following a contingency event come from the solution of a new problem.

E. Receding Horizon Planning

In a traditional security-constrained unit commitment and dispatch employed in a day-ahead market, each solution results in a set of hourly contracts for a 24-hour horizon, representing the operating plan for an entire day. Fig. 4 illustrates this approach for a "3-hour day" where stage 1 represents the day-ahead market, and stage 2 represents the real-time market, in this example, a single-period optimization run every half hour. In contrast, the proposed approach is intended to be used in a receding horizon scheme, as shown in Fig. 5. In this framework each run determines an optimal plan and corresponding contracts for the subsequent hour only (or several hours in the case of the commitment of slow-start units). It does so, however, in the context of a multi-period look-ahead. That is, the plan is optimal in the sense that it maximizes the total expected net benefits over the planning horizon subject to intertemporal constraints and costs, such as those related to ramping, storage and unit startup and shutdown. By solving frequently with updated input data (e.g. load and wind forecasts), the uncertainty associated with the operating plan for the next period is greatly reduced from the once-per-day approach, resulting in a more efficient solution.

While the stage 1 solutions in the proposed receding horizon scheme provide a coarse-grained, perhaps hourly, operating plan that respects hourly intertemporal constraints and costs, its complexity is excessive and unnecessary for the intra-hour real-time updates. A multi-period problem with a much

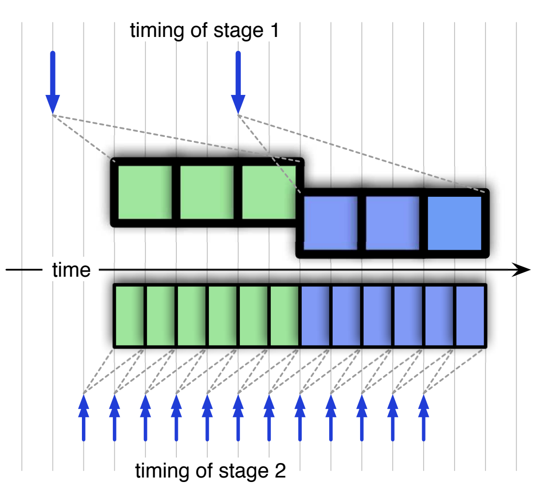

Fig. 4. Traditional Approach – Stage 1 runs *once-per-day*, finds hourly solution for *full day*; stage 2 runs intra-hour, finds single period solution subject to *day-ahead* contracts.

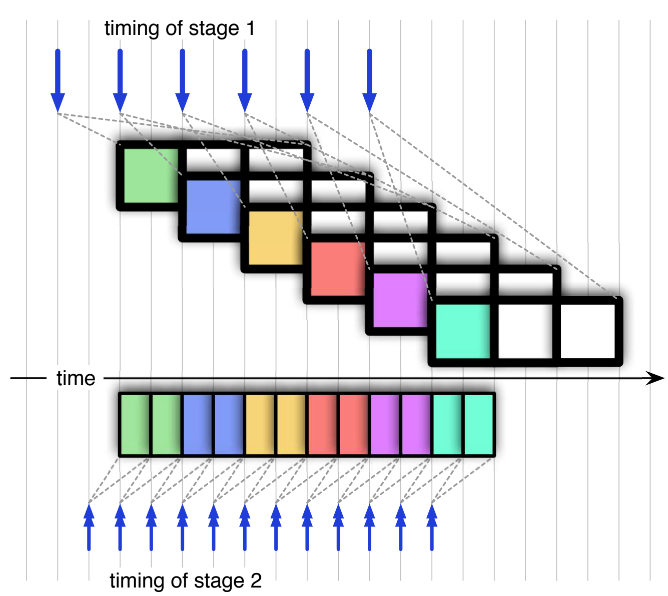

Fig. 5. Receding Horizon Approach – Stage 1 runs *hourly*, finds solution for *first hour* with hourly full-day look-ahead; stage 2 runs intra-hour, finds single period solution subject to *hour-ahead* contracts.

shorter horizon, or even an appropriately constrained single-period formulation is likely sufficient for this stage 2 problem. The majority of the emphasis in this paper is on the stage 1 problem, but it is important that the design of any stage 2 formulation be consistent with it.

III. Problem Formulation

A. Notation

In order to simplify the indexing notation, the index literals and their order are maintained wherever possible. No commas are used, so the superindex $tijk$ refers to time period t, generator i (or dispatchable load or storage unit i), base scenario j and contingency k. A dispatchable or curtailable load is modeled as negative generation, as in MATPOWER [16], each wind farm as a generator whose maximum output varies by scenario according to the forecast distribution, and a storage unit as a device with a given loss factor, energy capacity, and "charging" and "discharging" power capacities and efficiencies.

Δ Length of scheduling time slice in hours, typically 1 hour.

t Index over time periods.

T Set of indices of time periods in planning horizon, typically $\{1 \dots n_t\}$.

j Index over scenarios.

J^t Set of indices of all scenarios considered at time t.

k Index over post-contingency cases ($k = 0$ for base case, i.e. no contingency occured).

K^{tj} Set of indices of all contingencies considered in scenario j at time t.

i Index over injections (generation units, storage units and dispatchable or curtailable loads).

I^t Indices of all units (generators, storage and dispatchable or curtailable loads) available for dispatch in any contingency at time t.

I^{tjk} Indices of all units available for dispatch in post-contingency state k of scenario j at time t.

p^{tijk}, q^{tijk} Active/reactive injection for unit i in post-contingency state k of scenario j at time t.

$C_P^{ti}(\cdot)$ Cost function for active injection i at time t.

p_c^{ti} Active power contract quantity for unit i at time t.

p_+^{tijk}, p_-^{tijk} Upward/downward deviation from active power contract quantity for unit i in post-contingency state k of scenario j at time t.

$C_{P+}^{ti}(\cdot), C_{P-}^{ti}(\cdot)$ Cost for upward/downward deviation from active power contract quantity for unit i at time t.

r_+^{ti}, r_-^{ti} Upward/downward active contingency reserve quantity provided by unit i at time t.

$C_{R+}^{ti}(\cdot), C_{R-}^{ti}(\cdot)$ Cost function for upward/downward contingency reserve purchased from unit i at time t.

$\delta_+^{ti}, \delta_-^{ti}$ Upward/downward load-following ramping reserves needed from unit i at time t for transition to time $t+1$.

$C_{\delta+}^{ti}(\cdot), C_{\delta-}^{ti}(\cdot)$ Cost of upward/downward load-following ramp reserve for unit i at time t.

$C_\delta^i(\cdot)$ Quadratic, symmetric ramping cost on the difference between the dispatches for unit i in adjacent periods.

$\theta^{tjk}, V^{tjk}, p^{tjk}, q^{tjk}$ Vectors of voltage angles and magnitudes, active and reactive injections for power flow in post-contingency state k of scenario j at time t.

$g^{tjk}(\cdot)$ Nonlinear AC power flow equations in post-contingency state k of scenario j at time t.

$h^{tjk}(\cdot)$ Transmission, voltage and other limits in post-contingency state k of scenario j at time t.

$P_{\min}^{tijk}, P_{\max}^{tijk}$ Limits on active injection for unit i in post-contingency state k of scenario j at time t.

$Q_{\min}^{tijk}, Q_{\max}^{tijk}$ Limits on reactive injection for unit i in post-contingency state k of scenario j at time t.

$\delta_{\max+}^i, \delta_{\max-}^i$ Upward/downward load-following ramping reserve limits for unit i.

Δ_+^i, Δ_-^i Upward/downward physical ramping limits for unit i for transitions from base ($k = 0$) to contingency cases.

s_+^{ti}, s_-^{ti} Upper/lower bounds on the energy stored in storage unit i at the end of period t.

$S_{\max}^{ti}, S_{\min}^{ti}$ Stored energy max/min limits for storage unit i at time t.

S_0^i Initial stored energy (expected) in storage unit i.

$p_{\rm sc}^{tijk}, p_{\rm sd}^{tijk}$ Charge/discharge power injections of storage unit i in post-contingency state k of scenario j at time t.

$C_{\rm sc}, C_{\rm sd}$ Vectors representing contributions to the value of expected leftover stored energy in terminal states from charging/discharging.

$\eta_{\rm in}^i, \eta_{\rm out}^i$ Charging/discharging (or pumping/generating) efficiencies for storage unit i.

$\eta_{\rm loss}^i$ Fraction of stored energy lost per hour by storage unit i.

ψ^{tjk} Probability of contingency k in scenario j at time t (ψ^{tj0} is the probability of no contingency, i.e. the base case, for scenario j at time t).

α For contingency cases, the fraction of the time slice that is spent in the base case before the contingency occurs ($\alpha = 0$ means the entire period is spent in the contingency).

ψ_α^{tjk} Probability ψ^{tjk} of contingency k in scenario j at time t adjusted for α.

$$\psi_\alpha^{tjk} = \begin{cases} \psi^{tj0} + \alpha \sum\limits_{\kappa \in K^{tj} \neq 0} \psi^{tj\kappa}, & k = 0 \\ (1-\alpha)\psi^{tjk}, & \forall k \in K^{tj} \neq 0 \end{cases} \quad (1)$$

γ^t Probability of making it to period t without branching off the central path in a contingency in periods $1 \dots t-1$.

$\phi^{tj_2j_1}$ Probability of transitioning to scenario j_2 in period t given that we were at scenario j_1 in period $t-1$.

u^{ti} Binary commitment state for unit i in period t, 1 if unit is on-line, 0 otherwise.

v^{ti}, w^{ti} Binary startup and shutdown states for unit i in period t, 1 if unit has a startup/shutdown event in period t, 0 otherwise.

τ_i^+, τ_i^- Minimum up and down times for unit i in number of periods.

C_v^{ti}, C_w^{ti} Startup and shutdown costs for unit i at time t in \$ per startup/shutdown.

Individual variables can be grouped into vectors such as p^t for all active injections considered across all scenarios and contingencies at hour t and it will be consistent with the context. The subset referring to scenario j would be p^{tj}.

B. Formulation

The problem formulation can be expressed as a mixed-integer nonlinear optimization problem, where the optimization variable x is comprised of all the θ, V, p, q, p_c, p_+, p_-, r_+, r_-, δ_+, δ_-, $p_{\rm sc}$, $p_{\rm sd}$, s_+, s_-, u, v, and w variables. The last three are binary and the rest continuous. For simplicity, the formulation restricts the treatment of costs, deviations, ramping and reserves to consider only active power, but an extension to include reactive counterparts is straightforward.

1) Objective Function: The objective is expressed as the minimization of $f(x)$ which is comprised of six components.

$$f(x) = f_p(p, p_+, p_-) + f_r(r_+, r_-) + f_\delta(p) + f_{\rm lf}(\delta_+, \delta_-) + f_s(p_{\rm sc}, p_{\rm sd}) + f_{\rm uc}(v, w) \quad (2)$$

– cost of active power dispatch and redispatch

$$f_p(p, p_+, p_-) = \sum_{t\in T}\sum_{j\in J^t}\sum_{k\in K^{tj}} \psi_\alpha^{tjk} \sum_{i\in I^{tjk}} \Big[C_P^{ti}(p^{tijk}) + C_{P+}^{ti}(p_+^{tijk}) + C_{P-}^{ti}(p_-^{tijk})\Big] \quad (3)$$

– cost of contingency reserves

$$f_r(r_+, r_-) = \sum_{t\in T}\gamma^t \sum_{i\in I^t}\left[C_{R+}^{ti}(r_+^{ti}) + C_{R-}^{ti}(r_-^{ti})\right] \quad (4)$$

– cost of load-following ramping (wear and tear)

$$f_\delta(p) = \sum_{t\in T}\gamma^t \sum_{\substack{j_1\in J^{t-1}\\ j_2\in J^t}} \phi^{tj_2j_1} \sum_{i\in I^{tj_20}} C_\delta^i(p^{tij_20} - p^{(t-1)ij_10}) \quad (5)$$

– cost of load-following ramp reserves

$$f_{\rm lf}(\delta_+, \delta_-) = \sum_{t\in T}\gamma^t \sum_{i\in I^t}\left[C_{\delta+}^{ti}(\delta_+^{ti}) + C_{\delta-}^{ti}(\delta_-^{ti})\right] \quad (6)$$

– cost (or value, since it is negative) of expected leftover stored energy in terminal states

$$f_s(p_{\rm sc}, p_{\rm sd}) = -(C_{\rm sc}^{\mathsf T} p_{\rm sc} + C_{\rm sd}^{\mathsf T} p_{\rm sd}) \quad (7)$$

– startup and shutdown costs

$$f_{\rm uc}(v, w) = \sum_{t\in T}\gamma^t \sum_{i\in I^t}(C_v^{ti}v^{ti} + C_w^{ti}w^{ti}) \quad (8)$$

This minimization is subject to the following constraints, for all $t \in T$, all $j \in J^t$, all $k \in K^{tj}$, and all $i \in I^{tjk}$:

2) Standard OPF Constraints:

– nonlinear AC power balance equations

$$g^{tjk}(\theta^{tjk}, V^{tjk}, p^{tjk}, q^{tjk}) = 0 \quad (9)$$

– nonlinear transmission flow limits, voltage limits, any other OPF inequality constraints

$$h^{tjk}(\theta^{tjk}, V^{tjk}, p^{tjk}, q^{tjk}) \le 0 \quad (10)$$

3) Contingency Constraints:

– reserve, redispatch and contract variables

$$0 \le p_+^{tijk} \le r_+^{ti} \le R_{\max+}^{ti} \quad (11)$$

$$0 \le p_-^{tijk} \le r_-^{ti} \le R_{\max-}^{ti} \quad (12)$$

$$p^{tijk} - p_c^{ti} = p_+^{tijk} - p_-^{tijk} \quad (13)$$

– ramping limits on transitions from base to contingency cases

$$-\Delta_-^i \le p^{tijk} - p^{tij0} \le \Delta_+^i, \quad k \ne 0 \quad (14)$$

4) Intertemporal Constraints: The first of the contraints to spread intertemporally are the load-following ramp constraints, including ramping from a known dispatch at $t = 0$ into the first period.

– load-following ramping limits and reserves

$$0 \le \delta_+^{ti} \le \delta_{\max+}^{ti} \quad (15)$$

$$0 \le \delta_-^{ti} \le \delta_{\max-}^{ti} \quad (16)$$

$$p^{tij_20} - p^{(t-1)ij_10} \le \delta_+^{(t-1)i}, \quad j_1 \in J^{t-1}, j_2 \in J^t \quad (17)$$

$$p^{(t-1)ij_10} - p^{tij_20} \le \delta_-^{(t-1)i}, \quad j_1 \in J^{t-1}, j_2 \in J^t \quad (18)$$

– storage constraints

$$p^{tijk} = p_{\rm sc}^{tijk} + p_{\rm sd}^{tijk} \quad (19)$$

$$p_{\rm sc}^{tijk} \le 0, \qquad p_{\rm sd}^{tijk} \ge 0 \quad (20)$$

$$s_-^{ti} \ge S_{\min}^{ti}, \qquad s_+^{ti} \le S_{\max}^{ti} \quad (21)$$

$$s_\Delta^{tijk} \equiv -\Delta(\eta_{\rm in}^i p_{\rm sc}^{tijk} + \frac{1}{\eta_{\rm out}^i} p_{\rm sd}^{tijk}) \quad (22)$$

$$s_-^{ti} \le s_-^{(t-1)i} + s_\Delta^{tij0} - \Delta\frac{\eta_{\rm loss}^i}{2}\left(s_-^{ti} + s_-^{(t-1)i}\right) \quad (23)$$

$$s_+^{ti} \ge s_+^{(t-1)i} + s_\Delta^{tij0} - \Delta\frac{\eta_{\rm loss}^i}{2}\left(s_+^{ti} + s_+^{(t-1)i}\right) \quad (24)$$

$$S_{\min}^{ti} \le s_-^{(t-1)i} + \alpha s_\Delta^{tij0} + (1-\alpha)s_\Delta^{tijk}, \quad k \ne 0 \quad (25)$$

$$S_{\max}^{ti} \ge s_+^{(t-1)i} + \alpha s_\Delta^{tij0} + (1-\alpha)s_\Delta^{tijk}, \quad k \ne 0 \quad (26)$$

5) Unit Commitment:

– injection limits and commitments

$$u^{ti}P_{\min}^{tijk} \le p^{tijk} \le u^{ti}P_{\max}^{tijk} \quad (27)$$

$$u^{ti}Q_{\min}^{tijk} \le q^{tijk} \le u^{ti}Q_{\max}^{tijk} \quad (28)$$

– startup and shutdown events

$$u^{ti} - u^{(t-1)i} = v^{ti} - w^{ti} \quad (29)$$

– minimum up and down times

$$\sum_{y=t-\tau_i^+}^{t} v^{yi} \le u^{ti}, \qquad \sum_{y=t-\tau_i^-}^{t} w^{yi} \le 1 - u^{ti} \quad (30)$$

– integrality constraints

$$u^{ti} \in \{0,1\}, v^{ti} \in \{0,1\}, w^{ti} \in \{0,1\} \quad (31)$$

C. Discussion of Probability-weighted Costs

This section describes the assumptions regarding the probabilistic weighting of each term in the cost function. The planning horizon begins in some initial state at $t = 0$ with probability 1, where dispatches and storage unit states are known. From this initial state, transitions, each with some known probability, are possible to any of the scenarios considered for period $t = 1$. Thus, for $t = 1$, the sum of the probabilities assigned to each of the states is 1. The transition into the second period, however, is only possible provided that the system did not branch off into any contingency, i.e. the realized state is one of the base scenarios, at time $t = 1$. Since the sum of the probabilities of the base cases for $t = 1$ is less than 1, given the normal assumption that at least one contingency has a nonzero probability of occurrence, the probability of making it to $t = 2$ in the considered graph is also less than 1. So the probability of making it to period t without branching off the central path in a contingency can be written as follows.

$$\gamma^t = \sum_{j \in J^{t-1}} \psi^{(t-1)j0} = \sum_{j \in J^t, k \in K^{tj}} \psi^{tjk} < 1, \text{ for } t > 1 \quad (32)$$

The fact that the probabilities in future periods do not sum to 1 is consistent with $N-1$ contingency planning, where we choose to exclude the costs incurred following a contingency since it would involve exploration of the geometric possibilities of resuming normal operation afterward. In other words, no information is available about the effect of the decision variables on the costs of branches that have been trimmed. Therefore the impact of these costs are explicitly ignored by excluding them from the optimization.

The individual transition probability coefficients, which are used to weight the ramp wear and tear costs in (5), can be arranged into a transition probability matrix Φ^t where each column sums to 1. This given transition probability matrix for each period t relates the probabilities of base scenarios in $t-1$ to scenarios in t.

$$\begin{bmatrix} \sum_k \psi^{t1k} \\ \sum_k \psi^{t2k} \\ \vdots \\ \sum_k \psi^{tn_{J^t}k} \end{bmatrix} = \Phi^t \begin{bmatrix} \psi^{(t-1)10} \\ \psi^{(t-1)20} \\ \vdots \\ \psi^{(t-1)n_{J^{t-1}}0} \end{bmatrix} \quad (33)$$

IV. Solution Approaches

The problem formulation described above results in a large-scale, stochastic, non-linear, mixed-integer problem. This is obviously a formidable problem to solve, even for modestly sized networks with only a few scenarios and contingencies, and a solver for this full problem has not yet been implemented. While the focus of this paper is on the problem formulation, this section will describe some solution strategies that are being explored, highlighing approaches to some subproblems and useful approximations.

The basic strategy for the full non-linear, mixed-integer problem involves variable duplication and price-coordination between one or more continuous non-linear problems and a dynamic program, following the general approach used in [17].

Due to the sheer scale of the problem, even for modest sized systems the non-linear continuous subproblem (a multi-period, stochastic, contingency-based, security-constrained OPF) is intractable without further decomposition, such as described below. If, on the other hand, the nonlinear AC power balance and flow constraints are approximated by a linear DC model the continuous subproblem takes the form of a large, though still solvable, quadratic program (QP), eliminating the need for price coordination iterations. While this approximation sacrifices accuracy in the network model, it is still useful for testing other aspects of the proposed stochastic formulation. Likewise, the DC network version of the full problem can be posed as a mixed-integer quadratic program (MIQP) and solved by one of the commercially available state-of-the-art MIQP solvers.

A. Decomposition of Continuous Nonlinear Subproblem

In the design currently being tested, the decomposition is done by duplicating the variables representing all of the active power injections ($p^{tjk} \rightarrow \pi^{tjk}$). One set ($\pi^{tjk}$) is used in individual AC OPF problems, each representing a specific scenario or contingency in a specific period. The other set (p^{tjk}) is used in a central coordination quadratic program (QP) containing all of the intertemporal and inter-flow constraints and their corresponding variables and costs. With this decomposition scheme, the granularity on the level of a single OPF allows for massive parallelization and the central quadratic program, although large, can be solved for realistic-sized problems with the current generation of QP solvers. Coordination occurs only at the level of the two copies of the set of all active injections.

Essentially, all traditional OPF constraints (9)–(10) are imposed on the variables (θ^{tjk}, V^{tjk}, π^{tjk}, q^{tjk}), whereas constraints (11)–(26) and the original cost are imposed on variables (p, p_c, p_+, p_-, r_+, r_-, δ_+, δ_-, p_{sc}, p_{sd}, s_+, s_-), along with proxy constraints on system power balance used to limit the search space on p^{tjk}. Then, quadratic coordination costs arising from an augmented Lagrangian coordination scheme are imposed on the active injections to force equality of p^{tijk} and π^{tijk}. Thus the cost for each of the individual OPF subproblems comes entirely from the coordination costs

$$f_{\text{opf}}^{tjk}(\pi^{tjk}, \lambda^{tjk}) = \sum_i \left\{ \frac{b}{2}(\pi^{tijk})^2 + \left[\lambda^{tijk} - b\pi_{\text{old}}^{tijk} + c(\pi_{\text{old}}^{tijk} - p_{\text{old}}^{tijk})\right] \pi^{tijk} \right\} \quad (34)$$

where λ^{tijk} are the coordination multipliers, c is the coefficient of the quadratic augmentation term, so $c(\pi_{\text{old}}^{tijk} - p_{\text{old}}^{tijk})\pi^{tijk}$ is the linearization of this term about the previous iteration's values of (π^{tijk}, p^{tijk}), and b is the coefficient of the quadratic regularization term $\frac{b}{2}(\pi^{tijk} - \pi_{\text{old}}^{tijk})^2$; it has a damping effect in the trajectory of the coordination multipliers, a useful feature if problem costs are rather flat. Similarly the cost (2) (excluding the $f_{\rm uc}$ term) for the QP is modified to include coordination terms

$$f_{\rm coord}(p^{tjk}, \lambda^{tjk}) = \sum_t \sum_j \sum_k \sum_i \left\{ \frac{b}{2}(p^{tijk})^2 + \left[-\lambda^{tijk} - bp^{tijk}_{\rm old} + c(\pi^{tijk}_{\rm old} - p^{tijk}_{\rm old}) \right] p^{tijk} \right\} \quad (35)$$

An algorithm for solving the problem with this decomposition is as follows:

1. Initialize (λ^{tijk}), perhaps by solving the large QP without the coordination costs and with linearized network constraints; the (λ^{tijk}) take the values of the nodal prices at the active injection buses. This also assigns initial values to (p^{tijk}). Similarly, obtain initial values for (π^{tijk}), perhaps by solving each individual OPF with just the original fuel cost.
2. Update the coordination multipliers according to the mismatches, perhaps by
$$\lambda^{tijk} \leftarrow \lambda^{tijk} + \beta(\pi^{tijk} - p^{tijk}) \quad (36)$$
where β is a step-control parameter.
3. $\pi^{tijk}_{\rm old} \leftarrow \pi^{tijk}$ and $p^{tijk}_{\rm old} \leftarrow p^{tijk}$.
4. Solve one OPF for each of the flows considered in the problem, with cost (34) and restrictions (9)–(10) to obtain new (π^{tijk}).
5. Solve one large QP with cost (2) (modified to include $f_{\rm coord}$ and exclude $f_{\rm uc}$), constraints (11)–(26), and proxy network constraints such as
$$-0.1 P_D^{tijk} \leq P_D^{tijk} - \sum_i p^{tijk} \leq 0.1 P_D^{tijk} \quad (37)$$
where P_D^{tijk} is the system demand in flow $(tijk)$. From this, obtain new (p^{tijk}).
6. If any mismatch $\pi^{tijk} - p^{tijk}$ is larger than some tolerance, go to step 3 and repeat.

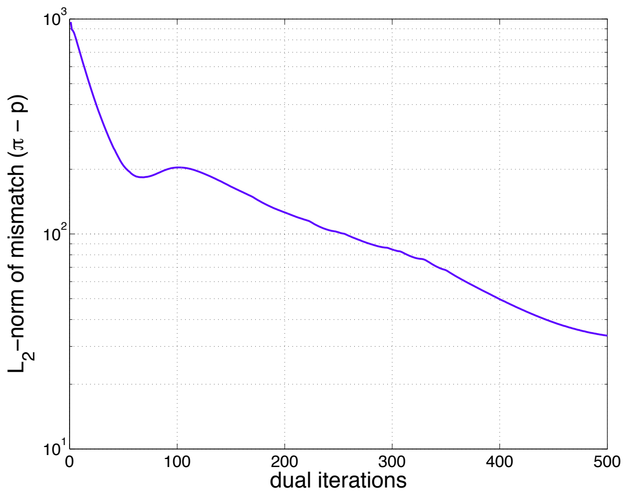

Fig. 6. Example convergence results for the non-linear subproblem illustrate the relatively slow convergence based on the L_2-norm mismatch.

V. Implementation Considerations

Implementing solution algorithms and developing appropriate test cases for the full formulation proposed above, in a receding horizon context with a second stage solver, is a massive undertaking that far exceeds the scope of a single paper. Our approach has been to build up the solvers and test cases piece-by-piece, implementing various approximations to the full formulation for testing different parts of the formulation.

For example, a version with an AC network model but without unit commitment is being used to refine the decomposition scheme described above. Convergence results for a 118-bus example are shown in Fig. 6, illustrating both the potential and the challenges involved in solving such a difficult problem. Similarly, a version with a DC network model and no problem decomposition is used to test the stochastic framework, the storage model and to develop wind input scenarios. This approach allows each aspect of the model to be refined somewhat independently on the way to an implementation of the full problem.

A. Problem Size

While it is beyond the scope of this paper to present comparative results here, the DC network implementation described above has been used to solve modest-sized problems. For example, a modified version of the IEEE 118-bus test system described in Table I is currently being used to explore the value of the stochastic framework for wind and storage. This case is essentially a 90,000-bus optimal power flow problem with numerous additional variables, costs and constraints and solves in approximately 19 minutes on a 12-core, 2.93 Ghz Intel Xeon machine. Detailed case studies and comparisons will be made in a follow-on paper.

B. Input Data

When developing test cases for a stochastic optimization tool such as this, it is crucial that the input data reflect the uncertainty characteristics faced by the system operator. This involves generating an appropriate set of scenarios, with corresponding transition probabilities, to represent the forecasted conditions that the operational plan should address. This must then be tested in a second stage on a corresponding set of realized trajectories that are statistically consistent with real-world forecasting errors. This section gives a brief overview of how this might be done for the case described in Table I,

TABLE I
Test Case Description

number of buses	118
number of generators	37
number of wind farms	11
number of grid-level storage facilities	4
number of curtailable loads	99
number of periods in horizon, $\|T\|$	24
number of scenarios per period, $\|J^t\|$	4
number of contingencies per scenario, $\|K^{tj}\| - 1$	7
number of variables in resulting QP	462,686
number of constraints in resulting QP	819,812

where the focus is on characterizing the uncertainty of wind and load. Similar care must be taken in the selection of the set of contingencies considered in each scenario.

The wind farms can be simulated based on data from the NREL-Eastern Wind Integration Study dataset [18]. A subset of data is used to represent a single class of day, selected to encompass realizations evolving from similar starting conditions and day ahead forecast trajectories. This replicates the knowledge that an ISO would have; knowing the season, the current conditions and a likely diurnal pattern of hourly forecasts for the coming day. Since actual forecast data is proprietary, and twenty-four hour forecast trajectories are not included in the NREL data, forecasts are simulated from observed data to provide appropriate accuracy and statistical distribution of errors [19], [20], and then classified into scenarios based on a k-means clustering algorithm [21]. The k-means algorithm is applied to reduce the simulated forecasts to four hourly scenarios including wind farm outputs and aggregate system load. Markov-based transition probabilities are estimated between each scenario in one hour and each scenario in the subsequent hour.

VI. Conclusions

A stochastic optimization framework has been presented for secure and efficient operations and planning of a bulk electric power system in the face of significant uncertainty. Security is handled through explicit inclusion of multiple scenarios with corresponding contingencies to represent both distributions of uncertain parameters such as wind outputs and low probability discrete events like generator and line outages. All of the standard OPF constraints are enforced for all of these states, each of which has its corresponding stochastic cost. Including the costs of involuntary load shedding provides a mechanism for establishing the value of reliability. The combination of the stochastic nature of the problem with the multi-period structure, with its intertemporal unit commitment, ramping and storage constraints, is handled in a way that incorporates a large number of possible trajectories while enforcing strict feasibility for a central path. The ability to schedule storage to optimally balance its varied potential uses is a feature of the proposed storage model. Detailed case studies demonstrating the unique features of this framework and comparing it with more conventional methods, along with the descriptions of the corresponding wind, storage and load models used as inputs, are beyond the scope of this paper and will be discussed in a future publication.

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