

# Fast Decoupled Power Flow for Unbalanced Radial Distribution Systems

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**Abstract** — This paper presents a novel power flow formulation and an effective solution method for general unbalanced radial distribution systems. Comprehensive models are considered including lines, switches, transformers, shunt capacitors, cogenerators, and several types of loads. A new problem formulation of three-phase distribution power flow equations taking into account the radial structure of the distribution network is presented. A distinguishing feature of the new problem formulation is that it significantly reduces the number of power flow equations, as compared with the conventional formulation. The numerical properties as well as the structural properties of distribution systems are exploited resulting in a fast decoupled solution algorithm. The proposed solution algorithm is evaluated on three-phase unbalanced 292-bus and 394-bus test systems with very promising results.

**Keywords** — power flow, load flow, radial network, distribution system, fast decoupled

## INTRODUCTION

Load flow is a very important and fundamental tool for the analysis of any power system and is used in the operational as well as planning stages. Certain applications, particularly in distribution automation and optimization of a power system, require repeated load flow solutions. In these applications it is very important to solve the load flow problem as efficiently as possible. Since the invention and widespread use of digital computers, beginning in the 1950's and 1960's, many methods for solving the load flow problem have been developed [1]. Most of the methods have "grown up" around transmission systems and, over the years, variations of the Newton method such as the fast decoupled method [2], have become the most widely used.

Unfortunately, the assumptions necessary for the simplifications used in the standard fast decoupled Newton method often are not valid in distribution systems. In particular,  $R/X$  ratios can be much higher. However, some work has been done to attempt to overcome these difficulties [3].

On the other hand, some of the methods based on the general meshed topology of a typical transmission system

are also applicable to distribution systems which typically have a radial or tree structure. Specifically, we will compare the proposed method to the standard Newton method [4], and the implicit  $Z_{bus}$  Gauss method [5, 6]. These methods do not explicitly exploit the radial structure of the system and therefore require the solution of a set of equations whose size is of the order of the number of buses.

Our goal was to develop a formulation and solution algorithm for solving load flow in large three-phase unbalanced systems which exploits the radial *topological structure* to reduce the number of equations and unknowns and the *numerical structure* to further reduce computation as in the fast decoupled methods for transmission systems.

Some algorithms specific to radial systems have been presented in [7]-[13]. These methods are all based on the concept of doing backward and/or forward sweeps of a ladder network. In [7], the backward sweep involves the computation of a driving point impedance equivalent at each bus and the forward sweep is used to update voltages and currents. The method proposed in [8] updates voltages and currents during the backward sweep and the forward sweep is a direct voltage correction. Various combinations of voltage, current, and power flow updates are used in the backward and forward sweeps of [9]-[14]. A fast decoupled Newton update, similar to the one proposed in this paper, was used for the backward sweep in [13].

In this paper we propose a new problem formulation and solution algorithm which has a flavor of several of the above methods. It is probably most closely related to the methods presented in [9] and [13]. Our numerical results also include a comparison with the back/forward sweep approach of [9].

### Basic Concept

Our approach is based on the fact that, given the voltage and current at one end of a distribution feeder and the currents injected into each sub-lateral, it is possible to compute all voltages and currents in the rest of the feeder. Since the current at the end of the feeder is zero, the end voltage can be taken as the unknown. And since the voltage at the source is specified we have a boundary condition we can use to compute a voltage mismatch.

Exploring the numerical characteristics of a distribution line, we can make decoupling approximations to the Jacobian used to update the unknown end voltages. This results in a very fast Newton-like solution to a reduced set of load flow equations.

For comparison, this fast decoupled algorithm was implemented along with the standard Newton method, the implicit  $Z_{bus}$  Gauss method, and the back/forward sweep method of [9]. Each method was evaluated and compared on a 292-bus and a 394-bus test system.

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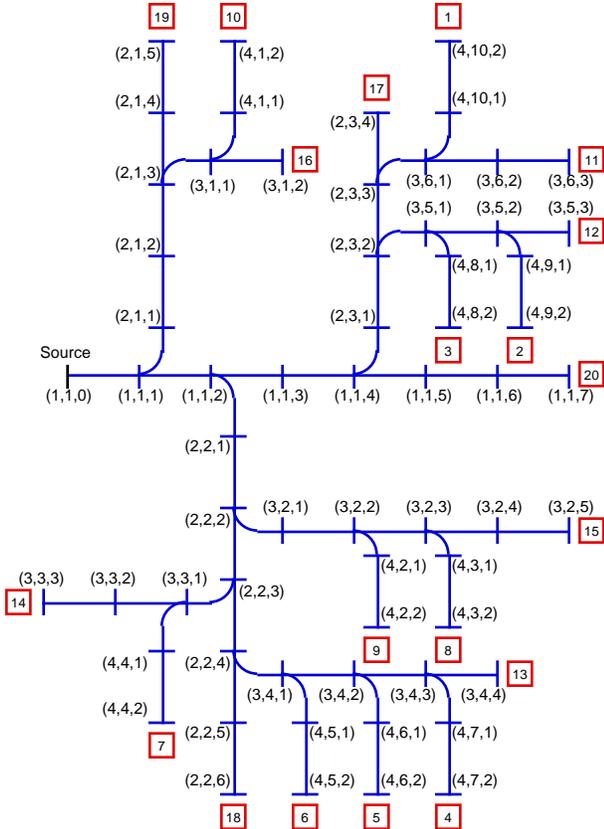
## BUS & LATERAL INDEXING

In most typical load flow formulations, a set of equations and unknowns is associated with each bus in the network and these equations and unknowns are organized by a particular bus ordering. Due to the radial structure of the systems under consideration, the number of equations and variables can be reduced so that each set of equations and unknowns corresponds to an entire lateral instead of an individual bus. Our formulation therefore calls for an appropriate lateral indexing to order these equations and variables.

A radial system can be thought of as a main feeder with laterals. These laterals may also have sub-laterals, which themselves may have sub-laterals, etc. So first we define the *level* of lateral  $i$  as the number of laterals which need to be traversed to go from the end of lateral  $i$  to the source. For example, the main feeder would be level 1, its sub-laterals would be level 2, their sub-laterals level 3, etc.

The laterals within level  $l$  are indexed according to the order visited during a depth-first traversal of the network. Each lateral can be uniquely identified by an ordered pair  $(l, m)$  where  $l$  is the lateral level and  $m$  is the lateral index within level  $l$ .

Buses are also indexed within each lateral starting with the first bus on the lateral, so that each bus is uniquely identified by an ordered triple  $(l, m, n)$  where  $n$  is the bus index. So  $(l, m, n)$  refers to the  $n^{\text{th}}$  bus on the  $m^{\text{th}}$  level  $l$  lateral. The source is given an index of  $(1, 1, 0)$ . Figure 1 shows an example of this indexing scheme on a sample 63-bus system. The boxed numbers



**Figure 1** Example of Bus & Lateral Indexing

show the *reverse breadth-first* (RBF) ordering of the laterals found by sorting the lateral indices in reverse order, first by level, then by lateral index.

We will also use the following shorthand notation when  $i$  is an ordered pair referring to a lateral and  $k$  is an ordered triple referring to a bus. Lateral  $i-1$  refers to the parent of lateral  $i$ , and bus  $k-1$  refers to bus  $k$ 's parent bus. Bus  $k+1$  may also be used to refer to the bus following bus  $k$  on the same lateral. This notation is used in indexing voltages, currents, impedances, etc.

## SYSTEM MODELING

For the purposes of power flow studies, we model a radial distribution system as a network of buses connected by distribution lines, switches, or transformers to a voltage specified source bus. Each bus may also have a corresponding load, shunt capacitor, and/or cogenerator connected to it. The model can be represented by a radial interconnection of copies of the basic building block shown in Figure 2. The dotted lines from the cogenerator, shunt capacitor, and load to ground are to indicate that these elements may be connected in an ungrounded delta-configuration. Since a given branch may be single-phase, two-phase, or three-phase, each of the labeled quantities is respectively a scalar,  $2 \times 1$ , or  $3 \times 1$  complex vector. For the simplicity of presentation we will occasionally assume everything is three-phase, although both single and two-phase laterals are handled by our program.

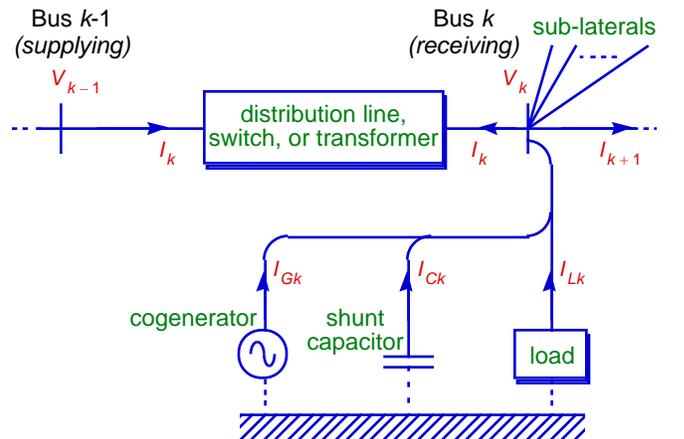
One of the key concepts behind our formulation is that the voltage and current at one bus can be expressed as a function of the voltage and current at the next bus. If we let

$$w_k = \begin{bmatrix} V_k \\ I_{k+1} \end{bmatrix} \quad (1)$$

we can write the branch update function as

$$w_{k-1} = g_k(w_k) \quad (2)$$

where  $w_k$  is a  $12 \times 1$  vector containing the real and imaginary parts of the voltages and currents at bus  $k$ . The func-



**Figure 2** Basic Building Block

tion  $g_k$  is determined by the sub-laterals attached at bus  $k$  as well as the models for distribution lines, switches, transformers, loads, shunt capacitors, and cogenerators.

From  $V_k$  we can compute the currents injected by the loads, shunt capacitors, and cogenerators from (4)-(8), according to the models presented below. Given  $I_{k+1}$  and the currents  $I_j$  injected into sub-laterals branching off from bus  $k$ , we apply KCL at bus  $k$  to give

$$I_k = I_{Gk} + I_{Ck} + I_{Lk} - I_{k+1} - \sum_{j \in A_k} I_j \quad (3)$$

where  $A_k$  is the set of buses adjacent to bus  $k$  on sub-laterals.

So for each bus  $k$  we can express the voltage and current at bus  $k-1$  as a function of the voltage and current at bus  $k$  in the form of (2) by using (3) and the appropriate equations from Table I.

#### Load Model

The load model used is a general model which allows each load to be either wye-connected or delta-connected and either constant impedance, constant current, or constant complex power. For the wye-connected case the injected currents can be computed as shown in (4)-(6). The bar denotes a constant value and the division in (4) and (6) is element-wise. Injected currents for delta-connected loads are computed by taking the differences of the appropriate elements in (4)-(6). This model could easily be generalized to be a linear combination of all of the above types.

#### Shunt Capacitor Model

Shunt capacitors are modeled as wye-connected or delta-connected constant admittance. The injected current as a function of voltage for the grounded wye-connected case is given in (7) where the multiplication is element-wise.

#### Cogenerator Model

Cogenerators are modeled as wye-connected or delta-connected constant complex power devices. The injected current given by (8) is therefore in the same form as (6).

#### Line Model

The line model used is the standard pi-model. The impedance of distribution line  $k$  is represented as a series impedance  $Z_k$ , and the line charging effects are divided between the two shunt arms, each with an admittance of  $Y_k/2$ . The impedance  $Z_k$  and the admittance  $Y_k$  are both  $n \times n$  complex matrices, where  $n$  is the number of phases in the line. The voltages and currents at the sending end of the line are computed via (9) and (10).

#### Switch Model

Sectionalizing switches are modeled as branches with zero impedance. The corresponding voltage and current relationships are given by (11) and (12).

#### Transformer Model

Three-phase transformers are modeled by the admittance matrix equivalent proposed in [15] for grounded-wye to grounded-wye connections.

**Table I** Update Equations

Element		Update Equation
load	constant Z	$I_{Lk} = V_k / \bar{Z}_{Lk}$ (4)
	constant I	$I_{Lk} = \bar{I}_{Lk}$ (5)
	constant PQ	$I_{Lk} = (\bar{S}_{Lk} / V_k)$ (6)
shunt capacitor		$I_{Ck} = \bar{Y}_{Ck} \cdot V_k$ (7)
cogenerator		$I_{Gk} = (\bar{S}_{Gk} / V_k)$ (8)
line		$V_{k-1} = V_k + Z_k \frac{1}{2} Y_k V_k - I_k$ (9)
		$I_k = \frac{1}{2} Y_k (V_k + V_{k-1}) - I_k$ (10)
switch		$V_{k-1} = V_k$ (11)
		$I_k = -I_k$ (12)
transformer		$V_{k-1} = (Y_k^{sp})^{-1} (I_k - Y_k^{ss} V_k)$ (13)
		$I_k = Y_k^{pp} V_{k-1} + Y_k^{ps} V_k$ (14)

$$\begin{bmatrix} I_k \\ I_k \end{bmatrix} = \begin{bmatrix} Y_k^{pp} & Y_k^{ps} \\ Y_k^{sp} & Y_k^{ss} \end{bmatrix} \begin{bmatrix} V_{k-1} \\ V_k \end{bmatrix} \quad (15)$$

From (15), we can solve for the voltage and current at the primary given the voltage and current at the secondary resulting in (13) and (14).

## PROBLEM FORMULATION

Here we present a new load flow formulation with a reduced number of equations and unknowns. The load flow problem is typically formulated as a set of non-linear power mismatch equations as functions of the bus voltages. The number of equations and unknowns in an  $n$ -bus all three-phase system is  $6(n-1)$ , since the source is the only voltage specified bus in a distribution system. Our formulation reduces the number of equations and unknowns to 6 times the number of laterals in the system. To illustrate our formulation we start with a system consisting of a single main feeder.

#### Single Feeder

Given the voltage and current at either end of the feeder we can compute the remaining voltages and currents. Note also that throughout this formulation currents could be replaced by complex power flows as in [13]. We have two boundary conditions, the current  $I_{n+1}$  at the end of the feeder is equal to zero, and the voltage  $V_0$  at the source is a specified constant  $\bar{V}_0$ . So we can take



**Figure 3** Single Feeder

either the current  $I_0$  at the source, or the voltage  $V_n$  at the end of the feeder to be the unknown. Unlike  $I_0$ , an initial guess to  $V_n$  is readily available without computation (balanced 1 p.u.), so we choose  $V_n$ , which we refer to as the *end voltage*, as the independent variable.

Using (2) and the fact that  $I_{n+1} = \mathbf{0}$  we can compute the source voltage (and current) as a function of  $V_n$ . Starting with  $w_n$  we apply the appropriate branch update function  $g_k$  consecutively until we reach  $w_0$ .

$$\begin{aligned} w_{n-1} &= g_n(w_n) = g_n\left(\begin{bmatrix} V_n \\ \mathbf{0} \end{bmatrix}\right) \\ w_{n-2} &= g_{n-1}(w_{n-1}) = g_{n-1} \cdot g_n\left(\begin{bmatrix} V_n \\ \mathbf{0} \end{bmatrix}\right) \\ &\vdots \\ w_0 &= \begin{bmatrix} V_0 \\ I_1 \end{bmatrix} = g_1(w_1) = g_1 \cdot \dots \cdot g_{n-1} \cdot g_n\left(\begin{bmatrix} V_n \\ \mathbf{0} \end{bmatrix}\right) \end{aligned} \quad (16)$$

We will denote the voltage part of the composite function  $g_1 \cdot \dots \cdot g_{n-1} \cdot g_n$  as  $\tilde{V}_0$ . At the solution the mismatch between the specified source voltage  $\tilde{V}_0$  and the computed source voltage  $\tilde{V}_0(V_n)$  must be zero. The power flow equations can therefore be written as

$$f(V_n) = \tilde{V}_0(V_n) - \tilde{V}_0 = \mathbf{0} \quad (17)$$

Since the voltage  $V$  is a complex  $3 \times 1$  vector this equation is equivalent to 6 real equations in 6 real unknowns independent of the number of buses on the feeder.

The Jacobian of  $f$  can be expressed using the chain rule and the branch Jacobians. The *branch Jacobian*  $G_k$  for bus  $k$ 's incoming branch is the Jacobian of equation (2)

$$G_k = \frac{g_k}{w_k} = \begin{bmatrix} \frac{V_{k-1}}{V_k} & \frac{V_{k-1}}{I_{k+1}} \\ \frac{I_k}{V_k} & \frac{I_k}{I_{k+1}} \end{bmatrix} \quad (18)$$

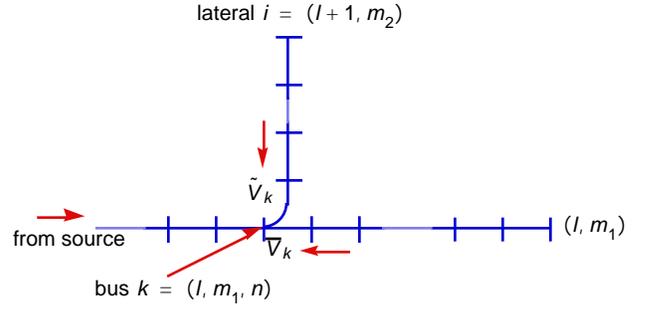
We express the system Jacobian for this simple single feeder case as

$$J(V_n) = \frac{f}{V_n} = \frac{\tilde{V}_0}{V_n} = \frac{\tilde{V}_0}{w_1} G_2 \dots G_{n-1} \frac{g_n}{V_n} \quad (19)$$

where the first term in the product is just the top half of  $G_1$  and the last term is the left half of  $G_n$ .

### General Radial Structure

To generalize this formulation to handle an arbitrary radial structure we first note that the voltage at the beginning of any lateral can be computed as a function of the end voltage *if* the currents injected into each sub-lateral are given. In a system with  $L$  levels, the level  $L$  laterals have no sub-laterals and can therefore be computed first. After all level  $L$  laterals have been computed, the currents injected into the sub-laterals of each level  $L-1$  lateral are known, hence the level  $L-1$  laterals can be computed. Next the level  $L-2$  laterals are computed, and so on, until the main feeder has been computed. This is the



**Figure 4** Mismatch Calculation

reverse breadth-first (RBF) ordering illustrated in Figure 1.

To complete the voltage mismatch calculation for each lateral we take the difference of the two voltages computed for each branching bus. Suppose lateral  $(l, m_1)$  has a level  $l+1$  sub-lateral  $i$  branching off at bus  $k$ , where  $i$  is the ordered pair  $(l+1, m_2)$  and  $k$  is the triple  $(l, m_1, n)$ . The mismatch for lateral  $i$  is taken as the difference between  $\tilde{V}_k$  as computed from the end voltage of sub-lateral  $i$ , and  $\bar{V}_k$  as computed from the end voltage of the supplying lateral  $(l, m_1)$ .

For lateral  $i$  starting at bus  $k$  we now have an equation similar to (17), but with  $V_n$  being replaced by a vector  $x$  containing the end voltages of all laterals.

$$f_i(x) = \tilde{V}_k(x) - \bar{V}_k(x) = \mathbf{0} \quad (20)$$

The function  $f_i$  does not depend on all elements of  $x$ , but only on the voltages of end buses supplied through bus  $k$  since they affect currents injected into the sub-laterals of lateral  $i$ . This relationship will be seen more clearly in Figure 5 when we look at the structure of the Jacobian.

Putting these equations together in RBF order we can express the new load flow equations compactly as

$$F(x) = \mathbf{0} \quad (21)$$

Note that  $\bar{V}_k(x)$  is no longer constant except when  $i$  is the main feeder and  $k$  is the source bus, i.e. the last set of equations when in RBF order.

Assuming we separate real and imaginary parts, for a system with  $m$  laterals, (21) is a set of  $6m$  non-linear equations in  $6m$  real unknowns. This is also assuming that all buses are three-phase; single and two-phase laterals would reduce these numbers accordingly.

### SOLUTION METHOD

The reduced set of load flow equations in (21) could be solved by any of several iterative methods for finding zeros of a set of general non-linear equations. The well-known Newton method proceeds as follows:

#### Newton Method

1. Choose an initial guess for the solution,  $x_0$ .
2. Set  $k = 0$ .
3. Evaluate  $F_k = F(x_k)$ .
4. Stop if  $\|F_k\|$  some tolerance.

5. Evaluate the Jacobian,  $J_k = \frac{F}{x} \Big|_{x_k}$ .
6. Solve  $J_k s_k = -F_k$ .
7. Let  $x_{k+1} = x_k + s_k$ .
8. Let  $k = k + 1$  and go to step 3.

As is typical in the Newton method, the majority of computation time is spent in step 5 evaluating the Jacobian, and step 6 solving for the update step. Any reasonable approximation which results in fewer computations in these two steps will greatly improve the speed of the algorithm, barring an increase in the total number of iterations.

First, we think of the Jacobian in terms of block elements corresponding to the laterals. The block element in block row  $i$  and block column  $j$  is the sensitivity of the voltage mismatch at lateral  $i$  with respect to variations in the end voltage of lateral  $j$ . Each block element of the system Jacobian can be expressed, using the chain rule, as a product of branch Jacobians along the path between the start bus of lateral  $i$  and the end bus of lateral  $j$ .

#### Numerical Structure of Branch Jacobian

The branch Jacobian  $G_k$  of (18) relates the sensitivity of voltages and currents at bus  $k - 1$  to small variations in the voltages and currents at bus  $k$ . For a three-phase branch it is a  $12 \times 12$  matrix. Due to the fact that for practical distribution lines the elements of the line impedance matrix  $Z_k$  and the line charging admittance  $Y_k$  are small with respect to voltage magnitudes, the branch Jacobian can be approximated by the identity matrix. To derive an analytical expression for each of the 144 terms of the matrix and analyze them to determine which terms can be neglected is obviously too tedious. So here we just give a motivation for the approximation.

A small change in voltage at bus  $k$  produces a correspondingly small change in the load, shunt, and cogenerator currents at bus  $k$ , as well as the series current in branch  $k$ . However, these currents are multiplied by the line's small impedance  $Z_k$ , making their effect on the voltage change at bus  $k - 1$  negligible. So from (9) we see that, neglecting the effects of the second term, a change in voltage at bus  $k$  produces nearly the same change in voltage at bus  $k - 1$ . This accounts for the upper left block of  $G_k$  in (18). Similarly, a small change in the current  $I_{k+1}$  affects only  $I_k$  and will have a negligible effect on the voltage  $V_{k-1}$  giving us approximately zero for the upper right block of  $G_k$ . Due to the small  $Y_k$  we see that small changes in  $V_k$  or  $I_{k+1}$  have very little effect on the first term in (10), accounting for the lower two blocks of  $G_k$  in (18).

Approximation of the branch Jacobian by the identity is the equivalent of replacing loads, shunt capacitors, and cogenerators by constant injected currents and neglecting line charging and line impedance. The same approximation is used for sectionalizing switches. For transformer sections, the upper left block of  $G_k$  is approximated by the identity divided by the tap ratio, and the lower right block by the identity times the tap ratio. In other words the transformer is approximated by its ideal equivalent. It is important to note that these approximations are only for simplifying the Jacobian used to compute the update step in Newton's method. They have *no* effect on the final solu-

tion, which is still based on the detailed models described in the section "SYSTEM MODELING".

#### Numerical Structure of System Jacobian

Taking into account the numerical structure of the branch Jacobians, we build up the system Jacobian via the chain rule. This leads to the special structure, dependent on the RBF ordering of the laterals, which is illustrated in Figure 5 for the sample system from Figure 1.

#### Decoupled Algorithm

Since the nonzero blocks below the block diagonal are nearly zero we can neglect them. By doing this we are essentially saying that the mismatch for lateral  $i$  is only affected by the end voltage of lateral  $i$  and the end voltage of lateral  $i$ 's parent.

This approximation greatly speeds up the solution of the update step in step 6 of the Newton algorithm. The matrix is now more sparse, but more importantly it is block-upper triangular and the update step can therefore be solved by block back-substitution.

#### Fast Decoupled Algorithm

In the decoupled algorithm, the evaluation of the Jacobian in step 5 is still quite complex and expensive to the extent of being impractical. Further approximations can be made by replacing the *near* identity blocks by an *exact* identity matrix and the *near* negative identity blocks by *exactly* negative identity. The exact identity blocks are actually divided by the appropriate tap ratios for blocks corresponding to laterals which include transformers.

The "Jacobian" is now a constant upper triangular matrix corresponding to the approximations made to the branch Jacobians as mentioned above. This means that

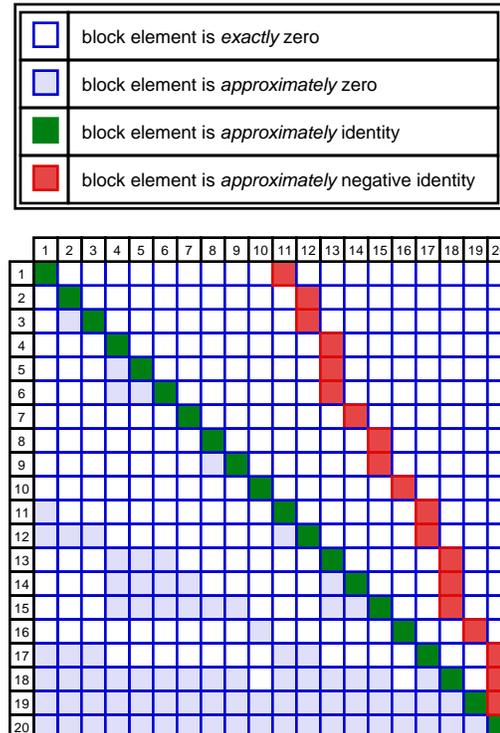


Figure 5 Structure of System Jacobian

step 5 has been essentially eliminated and step 6 is now a simple back-substitution. In fact, it is not even necessary to explicitly form and store the “Jacobian” since each row has a “1” on the diagonal and a “-1” in the column corresponding to the appropriate phase of the parent lateral. Here again, a “1” is replaced by the reciprocal of a corresponding transformer tap ratio where appropriate.

## RESULTS

The fast decoupled method described above was implemented in Matlab 4 along with the traditional formulation of the Newton [4] and implicit  $Z_{bus}$  Gauss methods [5] and a back/forward sweep method [9]. The implicit  $Z_{bus}$  Gauss method actually uses an optimally ordered, factored  $Y_{bus}$  instead of forming  $Z_{bus}$  explicitly.

The test systems are 292-bus and 394-bus unbalanced systems, each with 6 distribution transformers. According to our formulation, the first system has a total of 85 laterals and the second has 108 laterals, each including some single and 2-phase laterals. Three of the algorithms, excluding the back/forward sweep, require the solution of a large sparse system of linear equations. In the Newton and fast decoupled methods the resulting matrix is a Jacobian and in the implicit  $Z_{bus}$  Gauss method it is  $Y_{bus}$ . Table II shows the sizes and types of these matrices for our two test systems. If all buses were three-phase we would expect the dimension to be approximately  $6n$  for Newton,  $3n$  for implicit  $Z_{bus}$  Gauss, and  $6l$  for fast decoupled, where  $n$  is the number of buses and  $l$  is the number of laterals. Since there are some single and 2-phase buses the numbers are slightly smaller than these. Notice that in the fast decoupled method the matrix is triangular, that is, already in factored form. Unlike the first two methods, the proposed algorithm does not require the formation of  $Y_{bus}$  or the factoring of a large matrix.

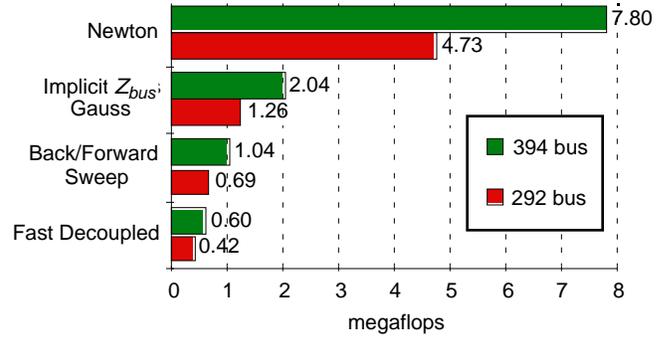
In the case of the 394-bus system, the number of real equations and unknowns, and therefore the size of the Jacobian, was reduced from 1996 for the traditional Newton formulation to 560 for the fast decoupled method. This decrease in size was accompanied by a correspondingly large decrease in the computation required to solve the load flow from a flat start. Figure 6 shows the computational complexity for each of the four methods in megaflops. These figures are based on the total number of floating point operations as reported by Matlab and should be indicative of run-time for the algorithms in compiled languages such as C or Fortran.

Our results show that the fast decoupled method is also more efficient than the back/forward sweep method based

**Table II** Sparse Matrix to be Factored

Algorithm	Type & Structure of Matrix	Dimension	
		292 bus	394 bus
Newton	real, general	1472	1996
Implicit $Z_{bus}$ Gauss	complex, symmetric	736	998
Back/Forward Sweep	<i>no matrix necessary</i>		
Fast-Decoupled	real, triangular <sup>a</sup>	426	560

<sup>a</sup>no factoring necessary



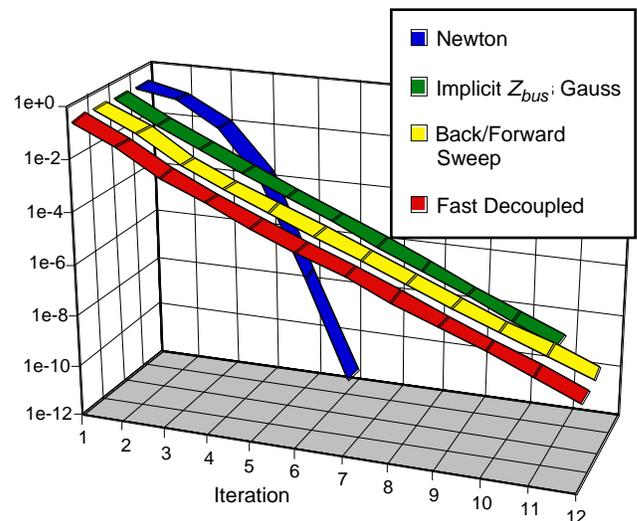
**Figure 6** Computational Complexity

on [9]. The difference is in the amount of computation necessary to update the end voltages. For the back/forward sweep approach this requires a full forward sweep calculating the voltage drops in each branch. In the fast decoupled approach it is a direct computation from the triangular “Jacobian”. Since the function evaluation in the proposed algorithm is equivalent to a backward sweep, we expect each iteration to require less computation.

In our tests, the Newton method demonstrated quadratic convergence as expected, while the other three methods converged linearly as shown in Figure 7.

## CONCLUSIONS & FUTURE WORK

In this paper we have exploited the radial structure (*physical property*) and the decoupling *numerical property* of a distribution system to develop a fast decoupled Newton method for solving unbalanced distribution load flow. It involves a reduced set of equations and unknowns proportional to the number of laterals in the network as opposed to the number of buses. Due to the reduced number of equations and the fact that the Jacobian is approximated by a constant triangular matrix, it is significantly faster than the implicit  $Z_{bus}$  Gauss method or the traditional Newton method based on  $Y_{bus}$ . Since each function evaluation involves updating each bus voltage and current



**Figure 7** Convergence for 394-bus System

and the “Jacobian” is triangular the computation in each iteration is proportional to  $n$ , making it suitable for very large radial systems. It is also shown to be more efficient than the back/forward sweep method of [9], due to the savings during the update of the end voltages.

The load flow method proposed in this paper could possibly be improved further in several ways. As described here, it is limited to *radial* systems with one voltage-regulated bus treated as the source. The authors believe that it could be generalized to handle weakly meshed systems using a compensation method similar to those presented in [10, 12, 14]. These approaches are based on choosing breakpoints to convert the system to a radial structure. The network is then solved using a radial load flow algorithm along with corrections to the breakpoint currents or powers. These approaches also allow for PV buses, treated as artificial breakpoints, making them suitable for weakly-meshed transmission systems. Since the methods in [10, 12, 14] are based on back/forward sweep, improvements might be possible by basing the radial network solver on the fast decoupled method proposed in this paper.

It may also be possible to extend this formulation to include distribution transformers of other connection types such as the remainder of those presented in [15].

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